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**ON SIMPLE AERODYNAMIC SENSITIVITY
DERIVATIVES FOR USE IN INTERDISCIPLINARY
OPTIMIZATION**

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Although both of the subject theories have shortcomings, they have been shown to give meaningful results in a variety of applications. For instance, low-aspect-ratio theory has been shown to give reasonable results for the steady state aerodynamic characteristics of slender configurations at transonic speeds,⁶ and for determining the static divergence characteristics of all moveable, delta-wing control surfaces.⁷ Piston theory has been used in flutter calculations of a variety of configurations.^{8,9} Indeed, piston theory has been routinely used in parametric aeroelastic analyses of advanced configurations, most recently in National Aero-Space Plane (NASP) applications.¹⁰ Even in instances where quantitative results may be inaccurate, trend data obtained by using these theories are usually correct.

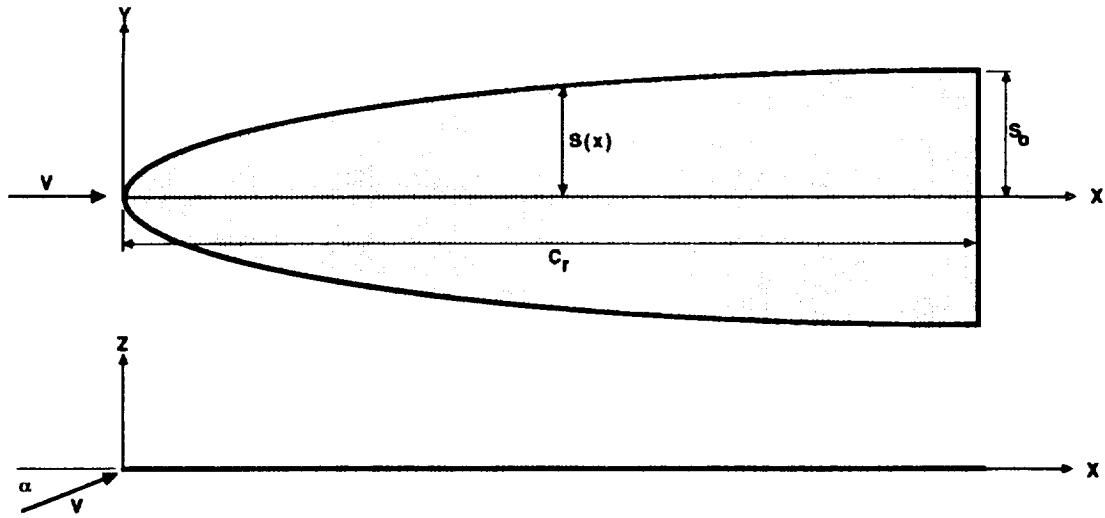
The present author believes that there is much to be learned by just going through the process of using aerodynamic sensitivity derivatives in multidisciplinary optimization. The knowledge gained from formulating and integrating aerodynamic sensitivity derivatives into the multidisciplinary process has significant value in and of itself. It is not necessary that the aerodynamic theory used be of "CFD quality", it is only necessary that the aerodynamic theory give reasonable results for the problem being addressed. Of course, in the long term, accurate CFD methods are needed before the full benefits of multidisciplinary optimizations can be realized in the design of future aerospace vehicles. Sobieski's plea is, therefore, still relevant and hopefully being heeded by CFD researchers. Nevertheless, it is hoped that the present exposition will make it clear that it is not necessary to await further aerodynamic theory developments before proceeding with the use of aerodynamic sensitivity derivatives in multidisciplinary optimization research.

AERODYNAMIC THEORIES

Two existing aerodynamic theories for which it is relatively simple and straightforward to

Low-Aspect-Ratio Theory

Planar wing, steady flow: - Low-aspect-ratio aerodynamic theory as used herein is applicable to planar, slender, pointed, symmetric wings such as that shown in Sketch 1. The shape of the wing is defined by the function $S(x)$ which is monotonically increasing in the stream direction. This requirement is necessary because portions of a wing downstream of the point of maximum span do not generate any lift. The root chord is denoted by C_r , and the semispan at the trailing edge by S_0 . The wing is inclined at an angle α to the free stream velocity V . The flow is considered to be two dimensional transverse to the flow direction. Compressibility effects are considered to be small.



Sketch 1

As developed by Jones³, the lifting pressure distribution in the spanwise direction y at streamwise station x for a wing at angle of attack α is given by

$$p(x,y) = 2 \alpha \rho V^2 \frac{S(x)}{\sqrt{S(x)^2 - y^2}} \frac{dS(x)}{dx} = \alpha \rho V^2 \frac{d}{dx} \left[\sqrt{S(x)^2 - y^2} \right]. \quad (1)$$

The lift per unit length at streamwise station x is obtained by integrating equation 1 across the span. The result is

$$\frac{dL(x)}{dx} = 2\pi \alpha \rho V^2 S(x) \frac{dS(x)}{dx} = \pi \alpha \rho V^2 \frac{d}{dx} [S^2(x)]. \quad (2)$$

The familiar expression for the total lift on the slender pointed wing results from the streamwise integration of equation 2 from the wing apex, $x=0$, to the trailing edge, $x=C_r$. The resulting expression for the total lift is

$$L = \pi \alpha \rho V^2 \int_0^{C_r} \frac{d}{dx} [S^2(x)] dx = \pi \alpha \rho V^2 S_0^2. \quad (3)$$

The total lift depends only on the width (maximum span) of the wing and not on the area, nor does it depend on the detailed shape of the planform.

The aerodynamic moment about the leading edge may be obtained by multiplying equation 2 by x and then integrating the result over the chord from the wing apex to the trailing edge. The resulting expression for the moment is

$$M = \pi \alpha \rho V^2 \left\{ C_r S_0^2 - \int_0^{C_r} S^2(x) dx \right\}. \quad (4)$$

Equations 3 and 4 are the basic equations defining the steady aerodynamic lift and moment on a low-aspect-ratio wing of general planform.

These lift and moment equations can be made more planform specific, yet still retain quite a bit of generality, if the planform shape function is assumed to be of the form

$$S(x) = K x^n \quad (5)$$

where K is a constant and n is an integer number. Substituting this shape function into equations 3 and 4, respectively, yields for the lift

$$L = \pi \alpha \rho V^2 K^2 C_r^{2n}, \quad (6)$$

and for the moment

$$M = \pi \alpha \rho V^2 K^2 \left(\frac{2n}{2n+1} \right) C_r^{2n+1} = L \left(\frac{2n}{2n+1} \right) C_r. \quad (7)$$

For the special case of a triangular wing, $K = \tan\theta$ (The angle θ is the complementary angle to the sweep angle.) and $n=1$, the expressions for the lift and moment become, respectfully,

$$L = \pi \alpha \rho V^2 \tan^2\theta C_r^2, \quad (8)$$

and

$$M = \frac{2}{3} \pi \alpha \rho V^2 \tan^2\theta C_r^3 = L \left(\frac{2}{3} C_r \right) \quad (9)$$

It is readily seen that the center of pressure is at two-thirds of the root chord measured from the apex.

An interesting result is obtained if the lift for the triangular wing is converted to coefficient form by dividing equation 8 by the product of the dynamic pressure $\frac{1}{2} \rho V^2$ and the wing area $C_r S_0$. The resulting expression for the lift coefficient is

$$C_L = \frac{\pi}{2} AR \alpha \quad (10)$$

where the wing aspect ratio $AR = \frac{4S_0^2}{C_r S_0}$. The pitching moment coefficient can be obtained by dividing equation 9 for the moment by the dynamic pressure, wing area, and reference chord C_r . The resulting expression for the pitching moment coefficient is

$$C_M = \frac{\pi}{3} AR \alpha = \left(\frac{2}{3} \right) C_L. \quad (11)$$

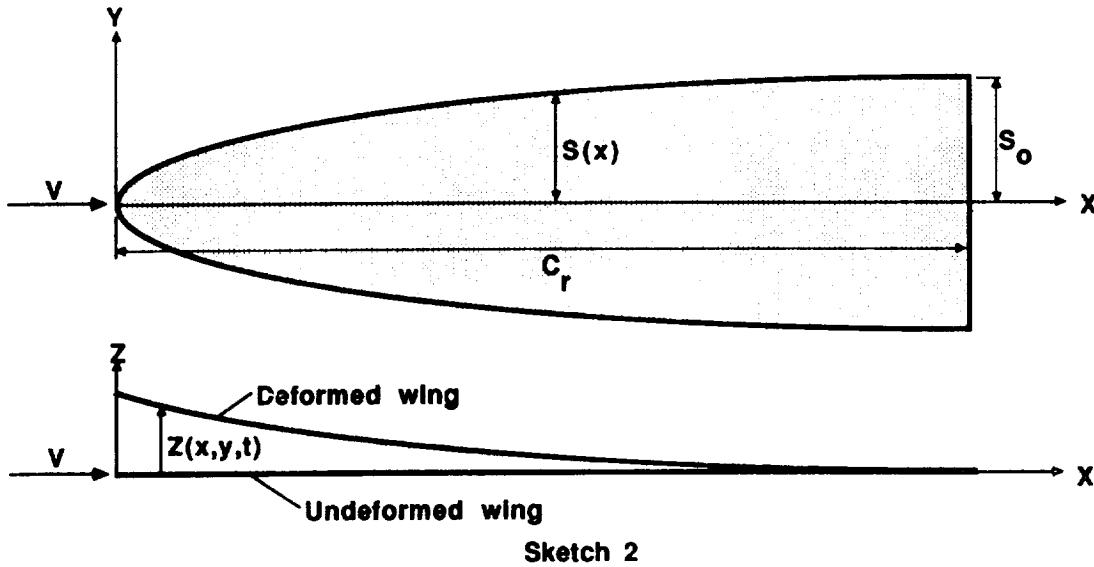
It is possible to develop aerodynamic sensitivity derivatives that are functions of selected planform variables by differentiating the appropriate equation(s) presented above. For example, the variation of total lift with maximum semispan (semispan at the trailing edge) can be obtained by differentiating equation 3. The result is

$$\frac{\partial L}{\partial S_0} = 2\pi \alpha \rho V^2 S_0. \quad (12)$$

Another example is to use equation 10 to determine the sensitivity of the lift coefficient to changes in aspect ratio.

Deformed wing, unsteady flow: - Garrick¹¹ extended low-aspect-ratio theory to the unsteady case of a deforming (nonplanar) wing. In discussing this extension, consider Sketch 2 that illustrates a slender wing that is deformed by bending out of the plane of the wing. The deformed shape is denoted by $Z(x,y,t)$. At a given chordwise station, the deformation is constant over the span. The normal wash $w(x,y,t)$ at a point x, y on the surface of a wing at an instance of time t is given by the relationship,

$$w(x,y,t) = -\frac{\partial Z(x,y,t)}{\partial t} - V \frac{\partial Z(x,y,t)}{\partial x}. \quad (13)$$



The spanwise pressure distribution at a particular chordwise station as derived by Garrick is given by

$$p(x,y,t) = -2\rho \left[\frac{\partial^2 Z(x,y,t)}{\partial t^2} + 2V \frac{\partial Z(x,y,t)}{\partial t \partial x} + V^2 \frac{\partial^2 Z(x,y,t)}{\partial x^2} \right] \sqrt{S(x)^2 - y^2} \\ - 2\rho V \left[\frac{\partial Z(x,y,t)}{\partial t} + V \frac{\partial Z(x,y,t)}{\partial x} \right] \frac{S(x)}{\sqrt{S(x)^2 - y^2}} \frac{dS(x)}{dx}. \quad (14)$$

Integration of this pressure distribution over the span gives the following result for the chordwise lift distribution.

$$\begin{aligned} \frac{\partial L(x)}{\partial x} = & -\pi \rho S^2(x) \left[\frac{\partial^2 Z(x,y,t)}{\partial x^2} + 2V \frac{\partial Z(x,y,t)}{\partial t \partial x} + V^2 \frac{\partial^2 Z(x,y,t)}{\partial x^2} \right] \\ & - 2\pi \rho V S(x) \frac{dS(x)}{dx} \left[\frac{\partial Z(x,y,t)}{\partial t} + V \frac{\partial Z(x,y,t)}{\partial x} \right]. \end{aligned} \quad (15)$$

Both the pressure and lift are functions of geometry, $S(x)$ and $\frac{dS(x)}{dx}$, the bending slope $\frac{\partial Z(x,y,t)}{\partial x}$ and curvature $\frac{\partial^2 Z(x,y,t)}{\partial x^2}$, the time rate of change of the bending slope $\frac{\partial^2 Z(x,y,t)}{\partial x \partial t}$, and the acceleration $\frac{\partial^2 Z(x,y,t)}{\partial t^2}$ and velocity $\frac{\partial Z(x,y,t)}{\partial t}$ of the bending deformations.

For the steady state case (Z not a function of time t) the pressure distribution is given by

$$\begin{aligned} p(x,y,t) = & -2 \rho V^2 \frac{\partial^2 Z(x,y)}{\partial x^2} \sqrt{S(x)^2 - y^2} - 2 \rho V^2 \frac{\partial Z(x,y)}{\partial x} \frac{S(x)}{\sqrt{S(x)^2 - y^2}} \frac{dS(x)}{dx} \\ = & -2 \rho V^2 \frac{\partial}{\partial x} \left[\frac{\partial Z(x,y)}{\partial x} \sqrt{S(x)^2 - y^2} \right], \end{aligned} \quad (16)$$

and the chordwise lift distribution is given by

$$\begin{aligned} \frac{\partial L(x)}{\partial x} = & -\pi \rho V^2 S^2(x) \frac{\partial^2 Z(x,y,t)}{\partial x^2} - 2\pi \rho V^2 S(x) \frac{dS(x)}{dx} \frac{\partial Z(x,y,t)}{\partial x} \\ = & -\pi \rho V^2 \frac{\partial}{\partial x} \left[\frac{\partial Z(x,y,t)}{\partial x} S^2(x) \right]. \end{aligned} \quad (17)$$

The total lift is obtained by integrating equation 17 from the leading edge apex to the trailing edge. The result is given by equation 18.

$$L = -\pi \rho V^2 S_o^2 \frac{\partial Z(C_r, 0)}{\partial x}. \quad (18)$$

Equation 18 is very similar to the expression for lift of the undeformed planar wing, equation 3, except now the angle of attack is the bending slope at the trailing edge. Thus, the total lift on a statically deformed low-aspect-ratio wing does not depend on the chordwise distribution of the deformed shape, but rather, only depends on the slope at the trailing edge of the wing.

The expression for the moment about the leading edge is given by

$$M = -\pi \rho V^2 \left\{ C_r S_o^2 \frac{\partial Z(C_r, 0)}{\partial x} - \int_0^{C_r} \frac{\partial Z(x, y, t)}{\partial x} S^2(x) dx \right\}. \quad (19)$$

This equation is very similar to the expression for the moment for the planar wing, equation 4.

The lift and moment for a deformed wing that is at an initial angle of attack α are simply the respective sums of equations 3 and 18 and of equations 4 and 19.

Aerodynamic sensitivity derivatives for the deformed low-aspect-ratio wing can be determined from the equations presented in this section in the same manner as pointed out for the planar wing case. For example, the change in chordwise lift distribution with span can be obtained from differentiating equation 15. It should be noted that in practical optimization problems the displacement related terms in equation 15 may be functions of the span and this fact must be taken into account in performing the required differentiations to determine the aerodynamic sensitivity derivatives. By way of example, consider equation 18 for the total steady lift which is a simpler expression than equation 15 but still can be used to illustrate the point. If the displacement is not a function of the span, then the change in total lift with respect to the maximum span is given by

$$\frac{\partial L}{\partial S_o} = -2\pi \rho V^2 S_o \frac{\partial Z(C_r, 0)}{\partial x}. \quad (20)$$

Now, if the displacement is a function of the span, the sensitivity becomes

$$\frac{\partial L}{\partial S_o} = -2\pi \rho V^2 S_o^2 \frac{\partial Z(C_r, 0)}{\partial x} - \pi \rho V^2 S_o \frac{\partial^2 Z(C_r, 0)}{\partial S_o \partial x}. \quad (21)$$

The first term in equation 21 is the same as equation 20. The second term results from the fact that the displacement is a function of the span.

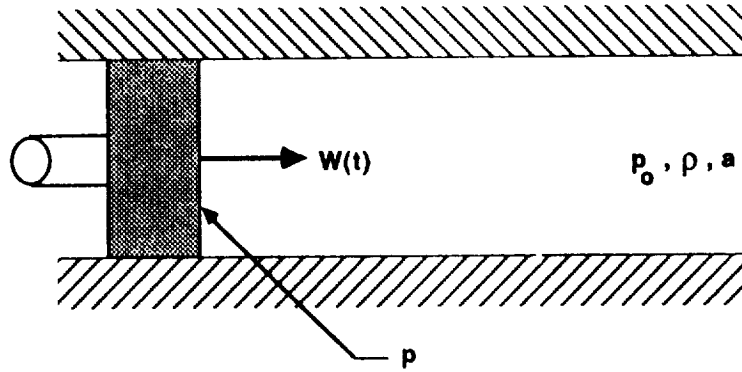
Piston Theory

Piston theory^{4,5} is a relatively simple supersonic aerodynamic theory. It has most often been used in supersonic flutter analyses, but it has steady state applications as well. Piston theory gets its name from the notion that the pressure at a point on an airfoil is related to the normal velocity of the airfoil in the same way that the pressure on the face of a piston moving in a channel is related to the motion of the piston. There are limitations on the use of piston theory. It is considered to be accurate when $M^2 \gg 1$ and when the product $M\delta$ is small. The term δ is the larger of the

maximum thickness-to-chord ratio of the airfoil section or the ratio of the dynamic displacement to airfoil section chord.

The relationship between the pressure p on the face of the piston and the motion of the piston $w(t)$ in a channel as illustrated in Sketch 3 is given by

$$p - p_0 = \rho a^2 \left\{ \left[\frac{w(t)}{a} \right] + \frac{(\gamma+1)}{4} \left[\frac{w(t)}{a} \right]^2 + \frac{(\gamma+1)}{12} \left[\frac{w(t)}{a} \right]^3 \right\} \quad (22)$$



Sketch 3

where p_0 is the pressure in the undisturbed fluid, a is the speed of sound in the fluid, and γ is the ratio of specific heats. The expression given by equation 22 is the third-order relationship. Typically only second-order piston theory is used in aerodynamic applications; the cubic term in equation 22 is not used. Thus, the second-order expression for the pressure becomes

$$p - p_0 = \rho a^2 \left\{ \left[\frac{w(t)}{a} \right] + \frac{(\gamma+1)}{4} \left[\frac{w(t)}{a} \right]^2 \right\}. \quad (23)$$

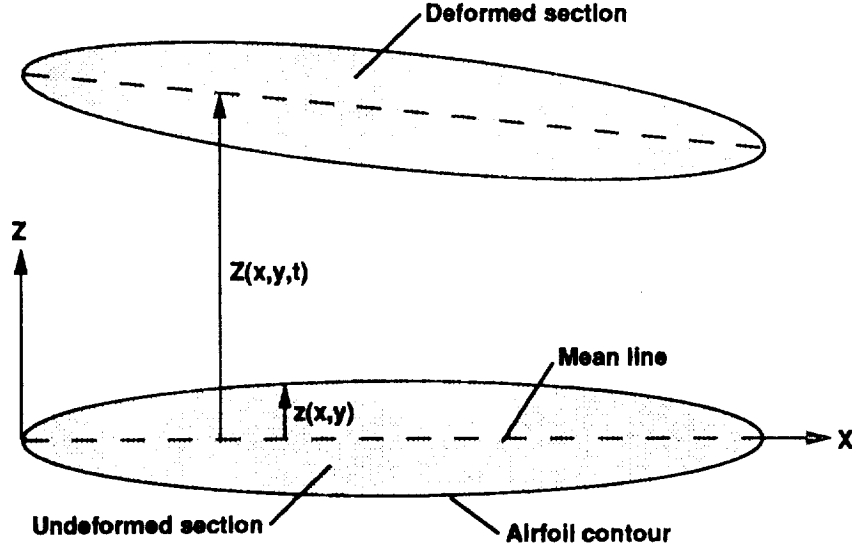
In aerodynamic applications, the piston motion term $w(t)$ in equation 23 is replaced by the normal wash of the airfoil. The normal wash on the upper surface of an airfoil surface is given by

$$w(x,y,t) = + \left(V \frac{\partial}{\partial x} + \frac{\partial}{\partial t} \right) Z(x,y,t) + V \frac{dz(x,y)}{dx} \quad (24)$$

where $Z(x,y,t)$ is the position of the mean surface of the airfoil and $z(x,y,t)$ is the function that describes the airfoil contour measured from the meansurface. These terms are illustrated in Sketch 4.

The corresponding normal wash on the lower surface is given by

$$w(x,y,t) = - \left(V \frac{\partial}{\partial x} + \frac{\partial}{\partial t} \right) Z(x,y,t) + V \frac{dz(x,y)}{dx}. \quad (25)$$



Sketch 4

The normal wash as given by equations 24 and 25 can be respectively substituted into equation 23 to provide expressions for the pressure on the upper surface and the lower surface of the airfoil. In most instances, however, it is the lifting pressure (difference between upper surface pressure and lower surface pressure) that is desired. By performing the two substitutions just mentioned and then taking the difference between the results, the lifting pressure is obtained. The resulting expression for the lifting pressure is

$$\Delta p(x,y,t) = -2 \rho a \left[1 + \Gamma \frac{\partial}{\partial x} z(x,y) \right] \left[\left(V \frac{\partial}{\partial x} + \frac{\partial}{\partial t} \right) Z(x,y,t) \right] \quad (26)$$

where $\Gamma = M \frac{(\gamma+1)}{2}$. The pressure at a point x,y is only dependent on the geometric characteristics of the airfoil section at that point and on the characteristics of the deformation at that point. There is no mutual interaction of a disturbance at one point on the pressure at another point.

It should be pointed out that there is an alternate form for equation 23 that follows from quasi-steady second-order linear supersonic theory.¹² This alternate form is

$$p-p_o = \rho a^2 \left(\frac{M}{\beta} \right) \left\{ \left[\frac{w(t)}{a} \right] + \left[\frac{(\gamma+1)M^4 - 4\beta^2}{4M\beta^3} \right] \left[\frac{w(t)}{a} \right]^2 \right\} \quad (27)$$

where $\beta = \sqrt{M^2 - 1}$. At high Mach numbers where $\beta \approx M$, this result approaches the second-order piston theory pressure given by equation 23. It is expected that the pressures given by equation 27 will be more accurate at lower supersonic Mach numbers than the pressure given by equation 23. The lifting pressure can be obtained by using the normal wash expressions given by equations 22 and 23 with the pressure equation 27. The resulting expression for the lifting pressure is

$$\Delta p(x,y,t) = -2\rho a \left[1 + \Gamma \frac{\partial}{\partial x} z(x,y) \right] \left[\left(V \frac{\partial}{\partial x} + \frac{\partial}{\partial t} \right) Z(x,y,t) \right] \quad (28)$$

but where now $\Gamma = \frac{(\gamma+1)M^4 - 4\beta^2}{2\beta^3}$. The form of this equation and equation 26 are the same, the only difference being in the value of the constant Γ .

The proceeding equations are put down for the unsteady case, but can be readily converted to the steady flow case by deleting the time dependent terms. That is, the deformation shape Z in equation 26 is not a function of time. The expression for the pressure in steady flow is

$$\Delta p(x,y) = -2 \rho a \left[1 + \Gamma \frac{\partial z(x,y)}{\partial x} \right] \left[V \frac{\partial Z(x,y)}{\partial x} \right] \quad (29)$$

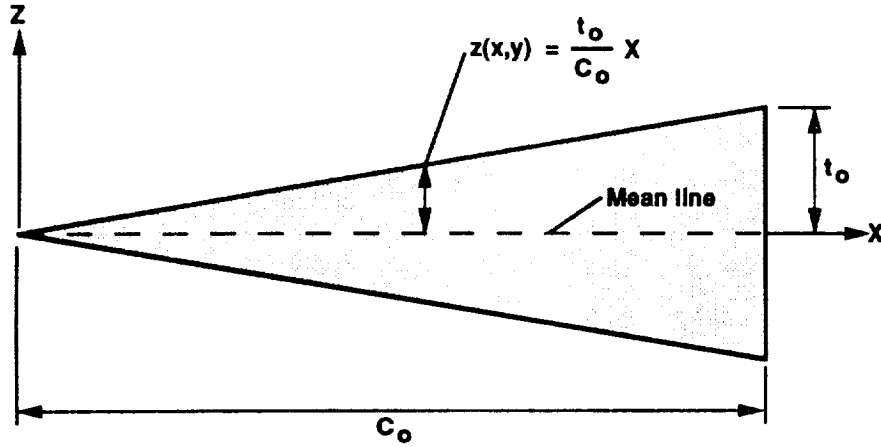
where Γ is one of the two values listed previously, depending on whether the second-order piston theory form or the second-order supersonic theory form is desired.

Because the surface pressures predicted by piston theory are a function of the airfoil section, this theory can be used to determine aerodynamic sensitivity derivatives that are functions of the airfoil section, for example, airfoil thickness-to-chord ratio. By way of illustration consider the simple delta airfoil illustrated in Sketch 5. The airfoil section is defined by the relationship

$$z(x,y) = \left(\frac{t_o}{c_o} \right) x. \quad (30)$$

The slope of the airfoil is given by

$$\frac{\partial z(x,y)}{\partial x} = \left(\frac{t_o}{c_o} \right). \quad (31)$$



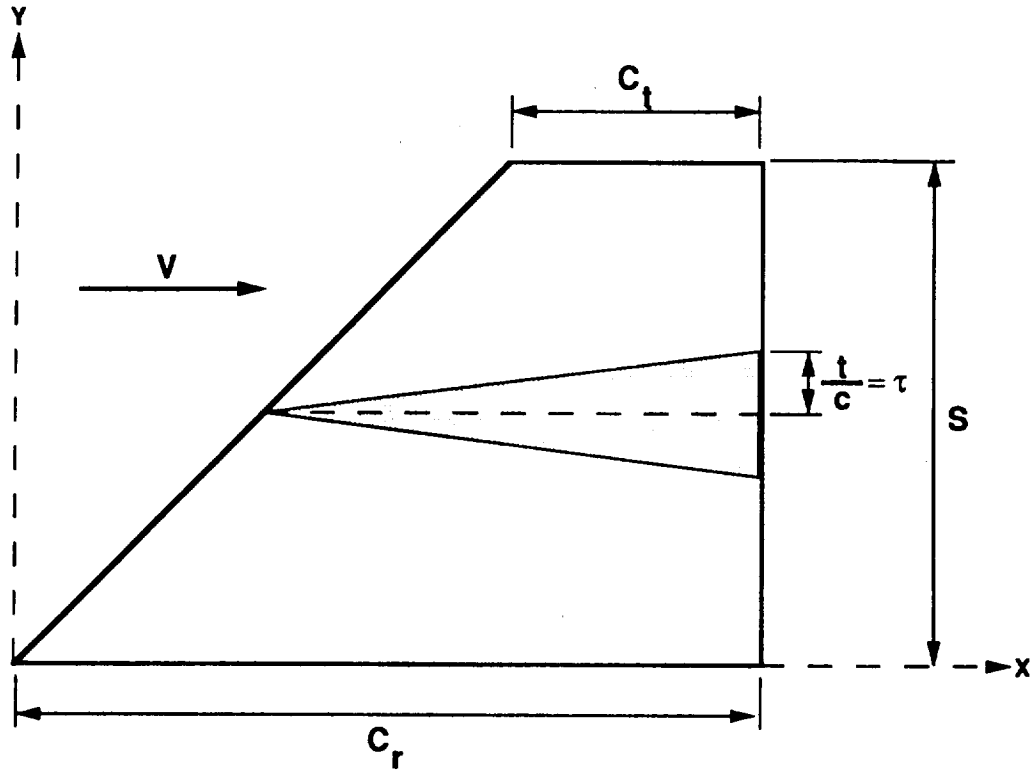
Sketch 5

After the value of the airfoil section slope given by equation 31 is substituted into equation 29 and the derivative with respect to the airfoil thickness is taken, the sensitivity of the pressure at point \bar{x}, \bar{y} to the maximum thickness is determined. This expression is

$$\frac{\partial(\Delta p(\bar{x}, \bar{y}))}{\partial t_o} = -2 \rho a \left[1 + \Gamma \left(\frac{1}{c_o} \right) \right] \left[\left(v \frac{\partial Z(\bar{x}, \bar{y})}{\partial x} \right) \right]. \quad (32)$$

Sensitivity of forces and moments with respect to other geometric variables such as area can also be obtained in a straightforward manner. To illustrate this, consider the clipped delta wing at constant angle of attack α that is illustrated in Sketch 6. The steady state lift for this configuration can be determined for the steady flow case by integrating the pressure given by equation 26 over the area of the planform, thus,

$$L = -2 \rho a \int_0^{\text{Span}} \int_0^{\text{Chord}} \left[1 + \Gamma \frac{\partial z(x,y)}{\partial x} \right] \left[v \frac{\partial Z(x,y)}{\partial x} \right] dx dy. \quad (33)$$



Sketch 6

For this example the airfoil thickness-to-chord ratio is a constant independent of span, so

$$\frac{\partial z(x,y)}{\partial x} = \left(\frac{t}{c} \right) = \tau. \quad (34)$$

Because the undeformed wing is at constant angle of attack α ,

$$\frac{\partial Z(x,y)}{\partial x} = -\alpha. \quad (35)$$

By using the relationships in equations 34 and 35 with the lift expression, equation 33, then the total lift becomes

$$L = 2 \rho a V \alpha \int_0^S \int_{x_{LE}}^{C_r} [1 + \Gamma \tau] dx dy \quad (36)$$

where $x_{LE} = \left(\frac{C_r - C_l}{S} \right) y$ for the trapezoidal wing shown in the sketch. After the integration is performed and terms collected, the total lift is given by

$$\begin{aligned} L &= 2 \rho a V \alpha \left[1 + \Gamma \tau \right] \text{Area} \\ &= \frac{1}{2} \rho V^2 \frac{4}{M} \alpha \left[1 + \Gamma \tau \right] \text{Area} \end{aligned} \quad (37)$$

where the ratio $\frac{4}{M}$ containing the Mach number M is the familiar linear supersonic lift curve slope. The sensitivity of the total lift with planform area for the trapezoidal wing used in this example is simply the derivative of equation 37 with respect to the area, or

$$\frac{\partial L}{\partial \text{Area}} = \frac{1}{2} \rho V^2 \frac{4}{M} \alpha \left[1 + \Gamma \tau \right] . \quad (38)$$

The sensitivity of aerodynamic quantities with respect to other geometric parameters may be determined by similar means. As was pointed out for low-aspect-ratio theory case, the deformation shape, either static or dynamic, may be a function of the geometric parameter so this fact must be taken into account in performing the required differentiations.

CANDIDATE PROBLEMS

As mentioned previously, there are classes of problems for which low-aspect-ratio theory and piston theory would be expected to give reasonably accurate results. In this section, a couple of optimization problems for their use are suggested. Of the two theories, piston theory is the most attractive for use because it is applicable to a greater variety of planforms and more suitable to be used in combined steady and unsteady applications.

The first problem is the design of a low-aspect-ratio all-moveable control fin that might be used on a missile. Both theories would be applicable to this problem. The objective would be to design a minimum weight fin that would have the planform size needed to provide the needed aerodynamic lift and moment authority to control the missile and yet have sufficient stiffness to satisfy a static divergence speed constraint of 1.15 times the limit speed. Of course, the structure would have to be strong enough to withstand the applied loads.

The second problem addresses the design of the wing for a supersonic transport airplane. Candidate planforms would be a clipped delta wing, or a clipped delta wing with cranks in either the leading edge or in the trailing edge or in both. Piston theory would be used for this application. The wing would be sized to meet some performance requirements, say to develop sufficient lift to

carry a prescribed payload. A flutter constraint could be applied as well as static deformation constraints. Aerodynamic sensitivity derivatives related to such geometric characteristics as airfoil section and planform area could be used. A minimum weight structure would be designed to satisfy the performance requirements within specified geometric, stress, and flutter constraints.

CONCLUDING REMARKS

Two existing aerodynamic theories that readily lend themselves to calculating certain aerodynamic sensitivity derivatives for use in multidisciplinary optimization studies have been reviewed briefly. The basic equations relating surface pressure (or lift) to normal wash was given and discussed in each case. Although no all-inclusive, rigorous developments of aerodynamic sensitivity derivatives were included, the exposition was taken to sufficient depth to provide the foundation needed by those wishing to use either of these theories to development aerodynamic sensitivity derivatives for use in multidisciplinary optimization studies. In addition, two sample problems are suggested in very general terms for studying the process of using aerodynamic sensitivity derivatives in optimization studies.

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13. ABSTRACT (Maximum 200 words) Low-aspect-ratio and piston aerodynamic theories are reviewed as to their use in developing aerodynamic sensitivity derivatives for use in multidisciplinary optimization applications. The basic equations relating surface pressure (or lift and moment) to normal wash are given and discussed briefly for each theory. The general means for determining selected sensitivity derivatives are pointed out. In addition, some suggestions in very general terms are included as to sample problems for use in studying the process of using aerodynamic sensitivity derivatives in optimization studies.				
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